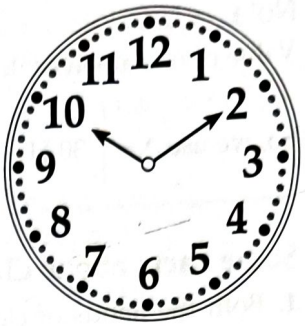
CLOCK



The figure on the right hand side is that of a clock. It has two hands. The shorter hand is the hour hand and the longer hand is the minute hand.

The face of the clock called dial is a complete circle having 360°. The dial is divided into 12 equal parts. These parts are marked as 1, 2, 3, ... , 11, 12. The angle between two consecutive numbers is $\frac{360°}{12}$ *i.e.* 30°. There are 5 equal small divisions between two successive numbers. Thus there are 60 small divisions in all. Each big division represents an hour and each small division represents a minute.

A clock hand turns through an angle of 360° in one revolution since complete dial is divided into 12 (big divisions) hours.

So, angle turned by an hour hand in 1 hour = $\frac{360°}{12} = 30°$

i.e. angular speed of hour hand = 30° per hour.

Also, angle turned by an hour hand in 1 minute = $\frac{30°}{60} = \frac{1}{2}^{\circ}$.

So, angular speed of hour hand = $\frac{1}{2}^{\circ}$ per minute.

Since complete dial is divided into 60 (small divisions) minutes.

So, angle turned by minute hand in 1 minute = $\frac{360°}{60} = 6°$

i.e. angular speed of minute hand = 6° per minute.

Hence, the relative angular speed of minute hand with respect to hour hand = $6° - \frac{1}{2}^{\circ}$ *i.e.* 5.5° per minute.

Relation between time and angle between the hands of clock

We know that angle turned by hour hand in 1 hour = 30°.

So, angle turned by hour hand in H hours = 30° × H = (30H)°.

Also, angle turned by hour hand in 1 minute = $\frac{1}{2}^{\circ}$.

So, angle turned by hour hand in M minutes = $\left(\frac{1}{2}M\right)^{\circ}$.

∴ Angle turned by hour hand in H hours M minutes = $\left(30\,H + \frac{1}{2}M\right)^{\circ}$.

Now, angle turned by minute hand in 1 hour = 360° which is equivalent to 0°

and angle turned by minute hand in 1 minute = 6°.

So, angle turned by minute hand in M minutes = (6 M)°.

∴ Angle turned by minute hand in H hours M minutes = (0 × H + 6 M)° = 6 M°.

So, angle between hands of clock at H hours M minutes

$$A = \left(30\,H + \frac{1}{2}M\right)^{\circ} - 6\,M^{\circ}$$

$$\Rightarrow \quad A = \left(30\,H - \frac{11}{2}M\right)^{\circ}$$

Note

Value of A is always taken as positive, negative sign is ignored.

So, we use $A = \left|\left(30\,H - \frac{11}{2}M\right)^{\circ}\right|$.

Some Facts about Clock

1. Both the hands of clock meet (coincide) 11 times in 12 hours.
2. Both the hands meet after every $\frac{12 \times 60}{11}$ minutes *i.e.* $65\frac{5}{11}$ min. The meeting take place at 12 : 00; $1 : 5\frac{5}{11}$; $2 : 10\frac{10}{11}$; ... and so on.
3. Both the hands of clock are opposite to each other (at 180°) 11 times in 12 hours.
4. Both the hands of clock are at right angles (at 90°) to each other 22 times in 12 hours.
5. Any angle other than 0° and 180° between the two hands is made 22 times in 12 hours.

ILLUSTRATIVE EXAMPLES

Example 1. *At what time do the hands of the clock meet between 6 : 00 to 7 : 00?*

Solution. At 6 o'clock, the hour hand is at 6 and the minute hand is at 12. It means the angle between two hands of clock is 180°.

To catch the hour hand, the minute hand has to cover a relative distance of 180° at a relative speed of 5.5° per minute.

$\therefore$ The time required to coincide the hands $= \frac{180}{5.5}$ minutes $= \frac{180 \times 2}{11}$ minutes

$= 32\frac{8}{11}$ minutes

$= 32$ min 44 sec.

Hence, the hands of the clock coincide at 6 : 32 : 44.

Example 2. *At what time between 4 : 00 and 5 : 00 will the hands of clock be at right angles?*

Solution. At 4 o'clock, the hour hand is at 4 and the minute hand is at 12. It means the angle between two hands of clock is 120°.

The hands will be at right angles twice between 4 : 00 and 5 : 00. First time while minute hand approaching towards hour hand *i.e.* near 1 and second time after crossing the hour hand *i.e.* near 7.

For the first time: The minute hand has to cover a relative distance of 30°.

So, the time required $= \frac{30}{5.5}$ minutes $= \frac{30 \times 2}{11}$ minutes $= 5\frac{5}{11}$ minutes

$= 5$ minutes 27 seconds.

For the second time: The minute hand has to cover a relative distance of 210°.

So, the time required $= \frac{210}{5.5} = \frac{210 \times 2}{11}$ minutes $= 38\frac{2}{11}$ minutes

$= 38$ minutes 11 seconds.

Hence, the hands of the clock are at right angles at 4 : 05 : 27 and 4 : 38 : 11.

Example 3. *Find the angle between two hands of the clock at 7 : 20.*

Solution. Here, H = 7, M = 20.

So, angle between two hands of the clock at 7 : 20

$$A = \left|\left(30 \times 7 - \frac{11}{2} \times 20\right)^\circ\right|$$

$$= |(210 - 110)^\circ| = 100^\circ.$$

Example 4. *Find the angle between two hands of the clock at 3 : 40.*

Solution. Here H = 3, M = 40.

So, angle between two hands of the clock at 3 : 40

$$A = \left|\left(30 \times 3 - \frac{11}{2} \times 40\right)^\circ\right| = |(90 - 220)^\circ| = |-130^\circ| = 130^\circ.$$

Example 5. *Find the time between 3 and 4 p.m. when the angle between hour and minute hand is 50°.*

Solution. Given, H = 3, A = 50°

$$\because \quad A = \left|\left(30\,H - \frac{11}{2}M\right)^\circ\right|$$

$$\Rightarrow \quad 50^\circ = \left|\left(30 \times 3 - \frac{11}{2}M\right)^\circ\right| \Rightarrow 50 = \left|90 - \frac{11}{2}M\right|$$

$$\Rightarrow \quad 90 - \frac{11}{2}M = \pm 50$$

Taking +ve, sign, we have

$$90 - \frac{11}{2}M = 50$$

$$\Rightarrow \quad 40 = \frac{11}{2}M \Rightarrow M = \frac{80}{11} = 7\frac{3}{11}.$$

So, time is 3 hour $7\frac{3}{11}$ minutes *i.e.* 03 : 07 : 16

Taking –ve sign, we have

$$90 - \frac{11}{2}M = -50$$

$$\Rightarrow \quad 140 = \frac{11}{2}M \Rightarrow M = \frac{280}{11} = 25\frac{5}{11}.$$

So, time is 3 hours $25\frac{5}{11}$ minutes *i.e.* 03 : 25 : 27.

Example 6. *A clock gains 4 seconds in 3 minutes and was set right at 9 : 00 a.m. What time will it show at 11 : 00 p.m. on the same day?*

Solution. Given that the clock gains 4 seconds in 3 minutes

$\Rightarrow$ it gains 20 × 4 = 80 seconds in 20 × 3 = 60 minutes *i.e.* 1 hour

Now, from 9 : 00 a.m. to 11 : 00 p.m. on the same day time passed is 14 hours.

So, in 14 hours, the clock will gain 14 × 80 seconds

= 1120 seconds

= 18 min. 40 sec.

Hence, the clock will show 11 : 18 : 40 p.m. at 11 : 00 p.m.

Example 7. *A clock loses 5 seconds in 4 minutes and was set right at 7 : 00 a.m. What time will it show at 1 : 00 p.m?*

Solution. Given that, the clock loses 5 seconds in 4 minutes

$\Rightarrow$ it loses 15 × 5 = 75 seconds in 15 × 4 = 60 minutes *i.e.* 1 hour.

Now from 7 : 00 a.m. to 1 : 00 p.m. on the same day time passed is 6 hours.

So, in 6 hours, the clock will lose $6 \times 75 = 450$ seconds = 7 min 30 sec.

Hence, the clock will show 12 : 52 : 30 p.m. at 1 : 00 p.m.

Example 8. *A clock loses 5 minutes every hour and was set right at 10 : 00 a.m. on a Sunday. When will it show the correct time again?*

Solution. For clock to show the correct time again, it should lose 12 hours.

Given that, the clock loses 5 minutes in 1 hour

$\Rightarrow$ it loses 60 minutes *i.e.* 1 hour in 12 hours

$\Rightarrow$ it loses 12 hours in $12 \times 12 = 144$ hours

$\Rightarrow$ it loses 12 hours in $\frac{144}{24} = 6$ days.

Hence, it will show the correct time again at 10 : 00 a.m. on Saturday.

Example 9. *A clock gains 5 seconds in 2 minutes and was set right at 9 : 00 a.m. If it shows 2 : 30 in the afternoon on the same day. What is the correct time?*

Solution. Given that, the clock gains 5 seconds in 2 minutes

$\Rightarrow$ it gains $12 \times 5 = 60$ seconds *i.e.* 1 minute in $12 \times 2 = 24$ minutes

$\Rightarrow$ when incorrect clock moves 25 minutes, correct clock moves 24 minutes.

Now, from 9 : 00 a.m. to 2 : 30 p.m. on the same day the time passed by incorrect clock

= 5 hours 30 minutes = 330 minutes.

$\therefore$ When incorrect clock moves 330 minutes, correct clock moves

$$= \frac{24}{25} \times 330 \text{ minutes} = 316 \text{ minutes } 48 \text{ seconds}$$

= 5 hours 16 minutes 48 seconds

Hence, the correct time is 2 : 16 : 48 p.m.

EXERCISE 3.3

1. At what time do the hands of the clock meet between 4 : 00 and 5 : 00?
2. At what time between 7 : 00 and 8 : 00 will the hands of a clock coincide?
3. At what time between 8 : 00 and 9 : 00 will the hands of clock be at right angles?
4. At what time between 5 : 00 and 6 : 00 will the hands of clock be at right angles?
5. Find the acute angle between two hands of a clock at 10 : 10 a.m.
6. Find the angle between two hands of a clock at 4 : 20 p.m.
7. At what time between 4'O clock 5'O clock will the hands of a clock make an angle of 20°?
8. Find the angle between the hands of a clock when it struck 15 minutes past 7.
9. By 20 minutes past 5, how many degrees has the hour-hand turned through?
10. A watch gains 5 seconds in 3 minutes and was set right at 10 : 00 a.m. What time will it show at 10 : 00 p.m. on the same day?
11. A watch gains 3 minutes in 1 hour and was set right at 8 : 00 a.m. What time will it show at 9 : 30 p.m. on the same day?
12. A clock loses 5 seconds in 1 minute and was set right at 7 : 00 a.m. What time will it show at 3 : 00 p.m. on the same day?
13. A watch loses 5 minutes in 2 hours and was set right at 6 : 00 a.m. What time will it show at 6 : 00 a.m. next day?

14. A watch gains 5 seconds in 3 minutes and was set right at 7 a.m. If it shows 4 : 15 in the afternoon on the same day. What is the correct time?
15. A watch loses 3 seconds in 2 minutes and was set right at 8 : 00 a.m. If it show 9 : 00 in the evening on the same day, what is the correct time?
16. A watch loses 6 minutes every hour and was set right at 11 : 00 a.m. on a Monday. When will it show the correct time again?
17. A local train from Mumbai leaves every 40 minutes from station. When inquired by a passenger, the help desk execution informed that the train had already left 10 minutes ago. If this information was given at 10 : 15 a.m.:
    (*i*) At what time did the train leave the station?
    (*ii*) At what time will the next train leave the station?
    (*iii*) For how long will that man has to wait for the next train?
    (*iv*) Find the angle formed between the minute hand and the hour-hand when the passenger will board the next train.

Answers

1. 04 : 21 : 49 **2.** 07 : 38 : 11 **3.** 08 : 27 : 16
4. 05 : 10 : 55 and 05 : 43 : 38 **5.** 115° **6.** 10° **7.** 04 : 18 : 11, 04 : 25 : 27
8. 127.5° **9.** 10° **10.** 10 : 20 p.m. **11.** 10 : 10 : 30 p.m.
12. 02 : 20 p.m. **13.** 05 : 00 a.m. **14.** 04 : 00 p.m.
15. 09 : 20 p.m. **16.** 11 : 00 a.m. on Saturday
17. (*i*) 10 : 05 a.m. (*ii*) 10 : 45 a.m. (*iii*) 30 minutes (*iv*) 52.5°